1. Consider a particle which moves randomly on the vertices of a triangle: Whenever the particle is at vertex \( i \), it moves to its clockwise neighbor vertex with probability \( p_i \) and to the counterclockwise neighbor with probability \( q_i = 1 - p_i, \ i = 1, 2, 3 \).

   (a) Find the proportion of time that the particle spends at each of the vertices.

   (b) How often does the particle make a clockwise move that is then followed by one consecutive counterclockwise move?

2. Consider a \( M/M/1/c \) queue, i.e., Poisson arrivals at rate \( \lambda \), iid exponential service times with parameter \( \mu \), 1 server, and the buffer size \( c \).

   a) Find the steady-state probability distribution of the number of customers in the system.

   b) Calculate the mean number of customers in the system.

   c) Calculate the mean delay experienced by customers. Note: customers who arrive and find the buffer full will not be considered in the mean delay calculation.

3. Consider a \( M/M/m \) queue, i.e., Poisson arrivals at rate \( \lambda \), iid exponential service times with parameter \( \mu \), \( m \) servers where each server can serve at most one customer, and the infinite buffer size.

   a) Find the steady-state probability distribution of the number of customers in the system. What is the stability condition (for existence of steady-state distribution)?

   b) What is the probability that a customer arrives and has to wait in line to get service?

   c) What will happen to the answers of parts (a) and (b) if \( m \to \infty \). (This is called a \( M/M/\infty \) queue)