ELEN E6761: Communication Networks
Midterm Exam: Solutions

Oct. 21, 2014

Name

Question 1	10	
Question 2	30	
Question 3	40	
Question 4	20	
Total	100	

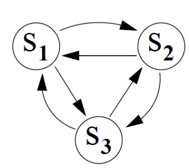
Please Read Carefully Before You Start:

- You are allowed to use one page formula sheet, however you should use your own formula sheet and are not allowed to exchange it with someone else's during the exam.
- The time limit is 1 hour and 30 minutes.
- If additional space is needed, use the back of the page for each problem.
- Show your work and clearly write all the steps. If you use a theorem or property, you should mention the name of the theorem or describe the property, otherwise you will not get full credit.

Version A

Question 1. (10 points) Circle the correct answer:

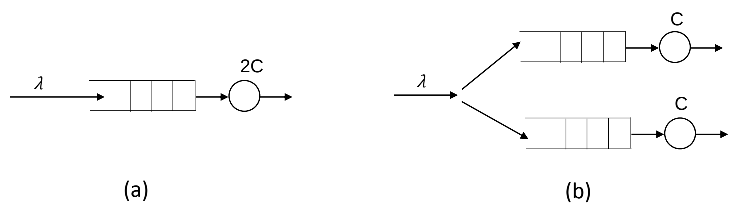
1. (2 points) The IP (routing protocol) corresponds to which layer of the protocol stack:
   (a) Physical layer
   (b) Link layer
   (c) Network layer
   (d) Transport layer
   (e) Application layer
   Answer: Network layer.
2. (2 points) WiFi in an standard for which layer of the protocol stack:
   (a) Physical layer
   (b) Link layer
   (c) Network layer
   (d) Transport layer
   (e) Application layer
   Answer: Link layer.
3. (2 points) Strategies such as FDMA and TDMA
   (a) are more suitable for bursty traffic than for constant rate traffic.
   (b) are more suitable for constant rate traffic than for bursty traffic. **Answer:** (b) Since FDMA and TDMA assign a sub-channel (a fixed fraction of time slots or bandwidth) to each user and do not change the schedule, they are more suitable for constant rate traffic.
4. (2 points) Consider a transmission line with capacity C. Suppose each user transmits at peak rate R for 90% of the time and remains silent 10% of the time. What is the best way to allocate the capacity among the users to accommodate more than C/R users?
   (a) circuit switching
   (b) packet switching
   Answer: Packet switching. Circuit switching accommodate C/R users, regardless of the users' activity pattern.
5. (2 points) What is the period of the following discrete-time Markov chain



(a) 1
(b) 2
(c) 3
(d) 4

Answer: (a) This Markov chain is aperiodic. period=$gcd\{2, 3, ...\} = 1$

Question 2. (30 points) Consider the following two systems when packets arrive as a Poisson process with rate λ and packet sizes are iid exponentially distributed with mean $\frac{1}{\mu}$ bits. In system (a), there is one (unlimited) buffer and all the packets are served by one server at rate $2C$ bits/second. In system (b), there are two (unlimited) buffers, each with a server that serves at rate C bits/second, and each arriving packet is randomly routed to one of the buffers with probability 0.5.



(i) (10 points) What is the stability condition of each system?

Solution:

(a) Packet service times are iid exponential random variables with mean $1/(2c\mu)$. The queue is M/M/1. We know that for such queue the stability condition is $\lambda < 2c\mu$.

(b) According to splitting property for Poisson process, packets arrive to each queue as Poisson process with rate $\lambda/2$. Then, each queue is again an M/M/1 queue, but with mean service time $1/(c\mu)$. So stability condition for both queues is that $\lambda/2 < c\mu$ or $\lambda < 2c\mu$.

(ii) (20 points) Calculate the average time that a packet spends in each system. Which system is better?

Solution: We use the formula for the mean delay of an M/M/1 queue.

(a)

$$\mathbb{E}\left[D\right] = \frac{1}{2c\mu - \lambda}$$

(b)

$$\mathbb{E}\left[D\right] = \frac{1}{2}\mathbb{E}\left[D_1\right] + \frac{1}{2}\mathbb{E}\left[D_2\right] = \mathbb{E}\left[D_1\right] = \frac{1}{c\mu - \frac{\lambda}{2}} = \frac{2}{2c\mu - \lambda}$$

Since delay in system (a) is less than delay in system (b), system (a) is better.

Question 3. (40 points) Consider a slotted ALOHA system with n backlogged stations and one receiver. Each station attempts a transmission in any time slot with probability q. Assume that the stations are backlogged all the time.

1. (10 points) Suppose the receiver can receive at most one packet in each time slot. What is the probability that it takes exactly k time slots for a transmission to be successful?

   Solution:

   Let A_k be the event that it takes exactly k time slots for a transmission to be successful. The probability that a transmission is successful in one time slot is $q(1-q)^{n-1}$. Thus

   $$\mathbb{P}(A_k) = q(1-q)^{n-1}\left(1 - q(1-q)^{n-1}\right)^{k-1}$$

   If you have solved the problem assuming that any successful transmissions, from any stations, would count, you would still get the full credit. In this case $\mathbb{P}(A_k) = nq(1-q)^{n-1}\left(1 - nq(1-q)^{n-1}\right)^{k-1}$

2. Now suppose the receiver is improved so that if no more than *two* stations transmit in the same slot, both packets are received correctly. Therefore collision takes place only if three or more stations transmit during the same slot.

   (a) (15 points) What is the throughput (r) as a function of n and q?

   Solution:

   $$\begin{aligned} r(q) &= 1 \times \mathbb{P}(\text{exactly 1 station transmits}) + 2 \times \mathbb{P}(\text{exactly 2 stations transmit}) \\ &= nq(1-q)^{n-1} + 2\frac{n(n-1)}{2}q^2(1-q)^{n-2} \\ &= nq(1-q)^{n-1} + n(n-1)q^2(1-q)^{n-2} \end{aligned}$$

(b) (10 points) What is the optimal q^* to maximize the throughput in part (a)?

Solution:

We take derivative of r(q):

$$\left.\frac{dr(q)}{dq}\right|_{q=q^*} = 0$$

$$n(1-q^*)^{n-1} - nq^*(n-1)(1-q^*)^{n-2} + n(n-1)2q^*(1-q^*)^{n-2}$$

$$- n(n-1){q^*}^2(n-2)(1-q^*)^{n-3} = 0$$

$$\Rightarrow (1-q^*)^2 + (n-1)q^*(1-q^*) - (n-1)(n-2){q^*}^2 = 0$$

$$\Rightarrow {q^*}^2\Big(-(n-1) - (n-1)(n-2) + 1\Big) + q^*(n-1-2) + 1 = 0$$

$$\Rightarrow {q^*}^2\Big(1 - (n-1) - (n-1)(n-2)\Big) + q^*(n-3) + 1 = 0$$

$$\text{for } n \le 2 : q^* = 1$$

$$\text{for } n > 2 : q^* = \frac{n - 3 + \sqrt{5n^2 - 14n + 9}}{2(n^2 - 2n)}$$

(c) (5 points) Suppose n is very large. Find an approximate expression for the maximum throughput of the system.

Solution:

For large n:

$$q^* \simeq \frac{(1+\sqrt{5})n}{2n^2} \simeq \frac{1+\sqrt{5}}{2n}$$

We also know that:

$$(1 - \frac{1+\sqrt{5}}{2n})^n \simeq e^{-\frac{1+\sqrt{5}}{2}}$$

Then we have:

$$r_{\max} = \frac{1+\sqrt{5}}{2}e^{-\frac{1+\sqrt{5}}{2}} + (\frac{1+\sqrt{5}}{2})^2 e^{-\frac{1+\sqrt{5}}{2}} = \frac{1+\sqrt{5}}{2}e^{-\frac{1+\sqrt{5}}{2}}(1 + \frac{1+\sqrt{5}}{2})$$

Question 4. (20 points) Consider a simple $M/M/1$ queue with arrival rate "λ" and mean service time "m".

(a) (10 points) What is the probability that an arriving customer does not have to wait in buffer in order to get service?

Solution:

By PASTA, the probability that customer sees the system empty is equal to probability that the system is empty. So:

$\mathbb{P}$ (customer does not have to wait in buffer) $= \pi_0$

For $\rho < 1$, system is stable and we know $\pi_0 = 1 - \rho = 1 - \lambda m$

For $\rho \geq 1$, system is unstable and $\pi_0 = 0$

(b) (10 points) Suppose the customer arrives at an empty queue. What is the probability that the customer leaves the system before the next customer arrives?

Solution:

T: be the time until the arrival of next customer $\sim \exp(\lambda)$

S: service time $\sim \exp(1/m)$

$$\mathbb{P}(S < T) = \frac{\frac{1}{m}}{\frac{1}{m} + \lambda} = \frac{1}{1 + \lambda m},$$

as you showed in the first problem of Homework 1.