

ELEN E6761: Communication Networks

Homework 7: Congestion Control & Resource Allocation Due: 12/15/2015

1. Consider sending a large file from a host to another over a TCP connection that has no loss. Suppose TCP uses AIMD for its congestion control without slow start. Assume constant round-trip time (RTT). (a) How long does it take for window size to increase from 6 packets to 12 packets (assuming no loss events)? (b) What is the *average* throughput (in terms of packets per RTT) for this connection during this time duration?
2. In the class, we derived a differential equation for the rate of TCP at each time and then found an expression for the average rate by considering the equilibrium. In this exercise, we will find the average throughput directly by considering the steady state (equilibrium) behavior.

Consider TCP in the congestion avoidance phase (AIMD) and in steady state. Then, we can approximately assume that the congestion window size varies from $W/2$ to W , and when it reaches W one packet loss happens.

(a) Show that the loss probability (fraction of packets lost) is equal to

$$q = \frac{1}{\frac{3}{8}W^2 + \frac{3}{4}W}$$

(b) Use the result above to show that if a connection has loss probability q , then its average rate is approximately given by

$$\cong \frac{1.22}{RTT\sqrt{q}}$$

3. Consider two users that share a single link using TCP. Suppose both users have the same RTT. In this case, as we saw in the class, when the link capacity and the link buffer size are large, TCP with AIMD behavior converges to both efficiency (full utilization of the link) and fair allocation (equal share). Suppose someone suggests a heuristic TCP with AIID behavior (i.e., additive increase, additive decrease). Does this TCP achieve efficiency? how about fairness?
4. Let x_r be the rate allocated to user r in a network where users' routes are fixed. Let $y_\ell = \sum_{r:\ell \in r} x_r$ be the total rate over link ℓ . Link ℓ is called a bottleneck link for user r if $\ell \in r$, and

$$y_\ell = c_\ell, \text{ and } x_s \leq x_r, \forall s : \ell \in s,$$

i.e., link ℓ is fully utilized and user r has the highest transmission rate among all users using link ℓ . Show that $\{x_r\}$ is a max-min fair rate allocation *if and only if* every source has at least one bottleneck link.

5. Consider a two-link, three user network as shown below. Link A has a capacity of 2 (packets/time-slot) and link B has a capacity of 1 (packet/time-slot). The route of user 1 consists of both links A and B ; the route of user 2 consists of only link A ; and the route of user 3 consists of only link B .

(a) Compute the resource allocations under the proportional fairness, minimum delay fairness, and max-min fairness. Assume corresponding weights are all one.

[Hint: For the max-min fair rate allocation, use the result of problem 4; and for the other two resource allocations, use Lagrange multipliers and the KKT theorem.]

(b) Write an expression for the distributed dual algorithm for each of the users for the case of proportional fairness.

