ELEN E6761: Communication Networks

Homework 5: Switch Due: 11/7/2017

1. In this exercise, we use an example to illustrate the throughput loss induced by HOL blocking. Consider a 2×2 switch, and assume input queues are infinitely backlogged. The destinations (output ports) of packets are randomly and uniformly chosen from $\{1, 2\}$ where 1 represents output port 1 and 2 represents output port 2. The destinations are independent across packets. Let $d_i(t)$ $(i = 1; 2)$ denote the destination of the head-of-line packet of input queue i at time t. When $d_1(t) \neq d_2(t)$; both head-of-line packets can be transferred to corresponding output ports; otherwise, one of them is randomly selected and transferred to the corresponding output port.

   a) Note that $d(t) = (d_1(t), d_2(t))$ is a Markov chain with state space $\{11; 12; 21; 22\}$. Write down the transition matrix of this Markov chain and compute the stationary distribution of the Markov chain.

   b) Based on the stationary distribution, compute the average throughput of this 2×2 switch. How much is the loss due to HOL blocking?

2. Consider a 2×2 VOQ switch. The arrival process into $VOQ(i, j)$ is Bernoulli with mean λ_{ij}: The arrival processes are independent across queues and time slots. Consider a scheduling policy that gives priority to edges $(1, 2)$ and $(2, 1)$; i.e., these edges are scheduled if they have any packets in their queues. Given λ_{12} and λ_{21}; compute the set of $(\lambda_{11}, \lambda_{22})$ that can be supported under the priority scheduling policy above, and the set of $(\lambda_{11}, \lambda_{22})$ that can be supported by the MaxWeight scheduling algorithm. Assume that $Q_{12}(0) = Q_{21}(0) = 0$. Is the priority scheduling algorithm throughput optimal?

3. Consider an $N \times N$ VOQ switch where the arrivals into queue (i, j) are Bernoulli with mean λ_{ij}: Assume that $\sum_h \lambda_{ih} < 1$ for every i and $\sum_k \lambda_{kj} < 1$ for every j . We have shown that the MaxWeight algorithm achieves optimal throughput (i.e., full capacity region). Instead, consider the following algorithm where the schedule at time slot t is chosen to be a matching that maximizes

$$\sum_{ij} M_{ij} Q^2_{ij(t)}$$

   In other words, we are now choosing the edge weights to be the square of the queue lengths. Show that this algorithm also achieves the full capacity region.

   Hint: Consider the Lyapunov function $V(Q(t)) = \sum_{ij} Q^3_{ij}(t)$.