ELEN E6761: Computer Communication Networks

Homework 2: Markov Chains, Queues, and ALOHA Due: 10/3/2017

1. Consider a particle which moves randomly on the vertices of a triangle: Whenever the particle is at vertex i, it moves to its clockwise neighbor vertex with probability p_i and to the counterclockwise neighbor with probability $q_i = 1 - p_i$, $i = 1, 2, 3$.

   (a) Find the proportion of time that the particle spends at each of the vertices.

   (b) How often does the particle make a clockwise move that is then followed by one consecutive counterclockwise move?

2. Consider a $M/M/1/c$ queue, i.e., Poisson arrivals at rate λ, iid exponential service times with parameter μ, 1 server, and the buffer size c.

   a) Find the steady-state probability distribution of the number of customers in the system.

   b) Calculate the mean number of customers in the system.

   c) Calculate the mean delay experienced by customers. Note: customers who arrive and find the buffer full will not be considered in the mean delay calculation.

3. Consider a $M/M/m$ queue, i.e., Poisson arrivals at rate λ, iid exponential service times with parameter μ, m servers where each server can serve at most one customer, and the infinite buffer size.

   a) Find the steady-state probability distribution of the number of customers in the system. What is the stability condition (for existence of steady-state distribution)?

   b) What is the probability that a customer arrives and has to wait in line to get service?

   c) What will happen to the answers of parts (a) and (b) if $m \to \infty$. (This is called a $M/M/\infty$ queue)

4. Assume that the number of packets n in a slotted ALOHA system at a given time is a Poisson random variable with mean $\alpha > 1$. Suppose we know the mean α and each packet is independently transmitted in the next slot with probability $1/\alpha$.

   (a) Find the probability that the slot is idle.

   (b) Suppose we observe an idle slot, show that the probability that there were n packets in the system, given this idle slot, is Poisson with mean $\alpha - 1$.

   (c) Find the probability that the slot is successful.

   (d) Suppose we observe a successful transmission, show that the probability that there are n remaining packets in the system, given this successful transmission, is Poisson with mean $\alpha - 1$.