ELEN E6761: Communication Networks

Homework 1: Poisson Processes & Markov Chains　　　　Due: 09/26/2017

1. Assume X and Y are two independent exponential random variables with rates λ and μ. Calculate $P(X \leq Y)$. *You might want to use this result in Problems 2 and 3.*

2. A person enters a supermarket and finds all of the two cashiers busy serving customers. There are no other customers in the supermarket, so the person will start service as soon as one of the customers in service leaves. Customers have iid exponentially distributed service times.

   (a) What is the probability that the person will be the last to leave the supermarket assuming that no other customers arrive?

   (b) If the average service time is 2 minutes, what is the average time the person will spend in the supermarket?

3. A machine works for an exponentially distributed time with rate μ and then fails. A repair crew checks the machine at times distributed according to a Poisson process with rate λ; if the machine is found to have failed then it is immediately replaced. Find the expected time between replacements of machines.

4. Prove the merging property of Poisson processes: If $N_1(t)$ and N_2 are independent Poisson processes with rate λ_1 and λ_2, then $N(t)$ is a Poisson process with rate $\lambda_1 + \lambda_2$ where $N(t) = N_1(t) + N_2(t)$. [Hint: consider one of the definitions of Poisson process for N_1 and N_2 and show that the definition also holds for N with rate $\lambda_1 + \lambda_2$].

5. Customers arrive at a bank according to a Poisson process with rate λ (customers/hour). Suppose two customers arrived during the first hour. What is the probability that
   (a) both arrived during the first 15 minutes?
   (b) at least one arrived during the first 15 minutes?

6. A certain town never has two sunny days in a row. Each day is classified as being either sunny, cloudy (but dry), or rainy. If it is sunny one day, then it is equally likely to be either cloudy or rainy the next day. If it is rainy or cloudy one day, then with probability 0.25 it will be the same the next day, otherwise it will be equally likely to be either of the other two possibilities. In the long run, what proportion of days are sunny? What proportion are cloudy?