# ELEN E6761: Communication Networks

## Homework 0: Probability Warm-up    Due: 09/12/2017

You need to be comfortable with elementary probability. This homework gives you an idea of the level of probability which is required in this course.

1. Let $Y$ be a Poisson random variable with mean $\mu > 0$ and let $Z$ be a geometrically distributed random variable with parameter $p$ with $0 < p < 1$. Assume $Y$ and $Z$ are independent.
   (a) Find $P(Y = i|Y < Z)$ for $i \geq 0$. Express your answer as a simple function of $p$, $\mu$ and $i$.
   (b) Find $E[Y|Y < Z]$ which is the expected value computed according to the conditional distribution found in part (a). Express your answer as a simple function of $\mu$ and $p$.

2. Let $X_1$ and $X_2$ be independent random variables, with $X_i$ being exponentially distributed with parameter $\lambda_i$. Find the pdf of $Z = \min\{X_1, X_2\}$.

3. Let $X$ and $Y$ be independent, exponentially distributed random variables with parameter $\lambda$. Find the pdf of $Z = |X - Y|$.

4. Let $X$ and $Y$ be independent Poisson distributed random variables with parameters $\lambda_1$ and $\lambda_2$ respectively. Show that $P(X = k|X + Y = n)$ is $B(n; p)$ (Binomial) where $p = \frac{\lambda_1}{\lambda_1+\lambda_2}$.

5. Find the mean and variance of random variables with the following characteristic functions: (a) $\Phi(u) = \exp(-5u^2 + 2ju)$ (b) $\Phi(u) = (e^{ju} - 1)/ju$.

6. Let $X$, $Y$ and $Z$ be random variables with finite expectations, and $a, b \in \mathbb{R}$, and $g : \mathbb{R} \to \mathbb{R}$. You need to be familiar with conditional expectation. Some of the properties of conditional expectation are the following. The first four are really trivial. You prove (iv)-(vii) assuming that random variables are discrete.

   (0) $E[X|a] = E[X]$.

   (i) $E(a|X) = a$.

   (ii) $E(aY + bZ|X) = aE(Y|X) + bE(Z|X)$ (*linearity of conditional expectation*)

   (iii) $E(X|Y) \geq 0$ if $X \geq 0$.

   (iv) $E(X|Y) = E(X)$ if $X$ and $Y$ are independent.

   (v) $E(X) = E(E(X|Z))$ (This is called *law of iterated expectation*)

   (vi) $E[Xg(Y)|Y)] = g(Y)E[X|Y]$, in particular $E[g(Y)|Y)] = g(Y)$

   (vii) $E[X|Y, g(Y)] = E[X|Y]$