ELEN E6761: Communication Networks
Final Exam

Fall 2014

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Please Read Carefully Before You Start:

- You are allowed to use one page formula sheet, however you should use your own formula sheet and are not allowed to exchange it with someone else’s during the exam.
- The time limit is 3 hours.
- If additional space is needed, use the back of the page for each problem.
- Show your work and clearly write all the steps. If you use a theorem or property, you should mention the name of the theorem or describe the property, otherwise you will not get full credit.

Version A
**Question 1.** Suppose that two TCP connections share a single congested link of speed 100 packets per second. Let $W_i(t)$ denote the window size of TCP $i$, at time $t$, $i = 1, 2$. At time 0, the initial window sizes are $W_1(0) = 1$ packet and $W_2(0) = 3$ packets. Suppose that both TCP connections are always in the congestion avoidance (Additive Increase-Multiplicative Decrease) and both have the same round trip time (RTT) of $0.1$ second. Further, the link has no buffer, i.e., if the total rate to the link exceeds the link speed, the packet drop happens and both senders are notified.

a) (10 points) What are the window sizes after 1 second?

*Note:* at any time in the table, if the window size is not integer, round up to the nearest integer, e.g., 1.5 is rounded up to 2.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
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<tr>
<td>$W_1(t)$</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W_2(t)$</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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b) (5 points) In the long run, will these two connections get about the same share of the bandwidth of the congested link?
**Question 2.** Consider the network shown below with indicated link costs. The complete paths from $W$ and $Y$ to $U$ (and between $W$ and $Y$) are not shown in the figure. $W$ has a minimum-cost path to $U$ of cost 5, and $Y$ has a minimum-cost path to $U$ of cost 6. All link costs in the network have strictly positive values. Nodes in the network are executing the distance-vector algorithm. Suppose the algorithm has converged.

![Network Diagram](image)

a) (5 points) Give the $X$’s distance vector for destinations $W$, $Y$, and $U$.

b) (5 points) Give a link-cost change for the link $(X,Y)$ such that $X$ will inform its neighbors of a new minimum-cost path to $U$.

c) (5 points) What is the smallest possible value of link-cost for $(X,Y)$ such that $X$ will *not* inform its neighbors of a new minimum-cost path to $U$?
Question 3. (20 points) Consider the following network with the indicated link costs. Use Dijkstra’s algorithm to compute the least-cost path from \( x \) to all the nodes. Show how the algorithm works by computing a table where each row corresponds to one iteration of the algorithm.
Question 4. (20 points) Consider a simple two-link, three-user network shown below. The link capacities and utility functions of the users are also given below. Compute the data transmission rates of the three users, $x_0$, $x_1$, and $x_2$, which maximize the sum network utility.

\[ U_0(x_0) = \log x_0; \quad U_1(x_1) = x_1; \quad U_2(x_2) = \log(1 + x_2) \]
**Question 5.** Consider the following $2 \times 2$ switch working based on Max Weight Matching $M^*(t)$, $t = 1, 2, \cdots$

![Diagram of a 2x2 switch]

a) (5 points) Consider the queues indicated in the figure and assume no further packets arrive. How many time slots does it take so that all the packets in the queues exit from the switch?

b) (15 points) Suppose the arrival rates $\lambda_{ij} = \frac{1}{4}$, $i = 1, 2; j = 1, 2$. Show that

$$\sum_{i,j} Q_{ij}(t)(\lambda_{ij} - M^*_{ij}(t)) \leq -f(Q(t)),$$

for a positive linear function $f$ which *only* depends on queues (no other variable or parameter must be present in $f$).
Question 6. (10 points) Suppose someone suggests a TCP congestion control algorithm that has the following differential equation

\[ \frac{dx_r(t)}{dt} = k_r \left[ 2(1 - q_r(t)) - x_r(t)q_r(t) \right], \]

where \( q_r(t) \) is the sum of the link prices on route \( r \), \( x_r(t) \) is the transmission rate of user \( r \) and \( k_r > 0 \) is some constant. Identify the utility function of user \( r \).

*Hint:* Recall the form of the primal congestion control algorithm, and compare it to the above differential equation.